Control Strategies for Retrial Queues with a Single Retrial Attempt

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Abstract—**In this paper, we consider retrial queueing systems with an input flow rate that depends on the number of calls in an orbit and with a limited number of retrials. According to threshold and hysteresis strategies, optimization problems for the effective operation of the system are formulated and solved. The quality functionals are built in terms of the stationary probabilities of the underlying Markov process describing the system. By solving the optimization problems, we obtain optimal policies that ensure effective and stable system operation under varying load conditions and retrial limitations.**

Keywords—*retrial queueing system; stationary probability; threshold strategy; hysteresis strategy*

I. Introduction

Retrial queues are a class of stochastic models that are applied for the modelling of computer and telecommunication systems, call centres and control systems in airports. These models consider the features of the service process that are not used in the classical queueing systems. If the input call finds all servers busy, it becomes a source of repeated calls. The calls can try to get service an infinite number of times [1], [2]. However, from a practical point of view, the number of repeated attempts is limited, particularly when security considerations are critical [6].

In this paper, we consider a multi-channel retrial queue with a controlled input flow rate and with a single retrial attempt to initiate service, where the input flow rate depends on the number of repeated attempt sources at a current moment (see [3]). This allows to control of the input flow to enhance service quality and achieve higher system revenue according to threshold and hysteresis strategies. The following parameters characterize the system: c is the number of channels, λ is the input flow rate, μ is the service rate and ν is the rate of repeated attempts.

II. Threshold Strategy

According to the threshold strategy, if at some moment the total number of customers in the system's orbit does not exceed the value H, the system operates in the first mode with the input flow rate h_1. If the number of customers exceeds H, the system switches to a second mode with an input flow rate h_2. Formally, setting a threshold strategy implies the following dependence on the number of repeated calls:

$$\lambda_j = \begin{cases} h_1, \ j <= H, \\ h_2, \ j > H. \end{cases}$$

Let us define this system as a two-dimensional continuous-time Markov process

$$Q(t,H) = (Q_0(t,H), Q_1(t,H)), \ t \geq 0$$

in the phase space $S(H) = \{0,1,...,c\} \times Z_+$, where $Q_0(t,H)$ is the number of busy channels at time t, $Q_1(t,H)$ is the number of repeated attempt sources (customers in orbit) at time t.

The optimal control problem for the retrial queues with a single retrial attempt consists in searching for such thresholds H that are a solution of the optimization problem:

$$L(H) = C_1 L_1(H) + C_2 L_2(H) - C_3 L_3(H) - C_4 L_4(H) - \\ -C_5 L_5(H) \to \max, \ H \in \{0, 1, ...\}, \qquad (1)$$

where the cost coefficients are: C_i is the income relating to the service of a call in the i-th mode, $i = 1,2$; C_3 is the penalty for refusing to provide a service; C_4 is the penalty relating to switching of the input flow rate; C_5 is the penalty relating to lost call after failed repeated attempt; $L_i(H) = \lim_{t\to\infty} t^{-1} L_i(t, H)$, $i = \overline{1,5}$: $L_i(t,H)$ is the number of calls served in the i-th mode, $i = 1,2$; $L_3(t,H)$ is the number of calls which become repeated calls; $L_4(t,H)$ is the number of switchings of the input flow; $L_5(t,H)$ is the number of calls that leave the system without service after repeated attempt. The quality functional $L(H)$ is built in terms of the system's stationary probabilities (see [3], [5]) via $L_i(t,H)$, $i = \overline{1,5}$:

$$L_1(H) = \mu \sum_{j=0}^{H} \sum_{n=1}^{c} n \pi_{nj}(H),$$

$$L_2(H) = \mu \sum_{j=H+1}^{\infty} \sum_{n=1}^{c} n \pi_{nj}(H),$$

$$L_3(H) = h_1 \sum_{j=0}^{H} \pi_{cj}(H) + h_2 \sum_{j=H+1}^{\infty} \pi_{cj}(H),$$

$$L_4(H) = h_1 \pi_{cH}(H) + (H+1)\nu \sum_{n=0}^{c} \pi_{nH+1}(H),$$

$$L_5(H) = \nu \sum_{j=1}^{\infty} j \pi_{cj}(H).$$

The solution to problem (1) is such threshold H that maximizes the average profit from the operation of the system.

III. Hysteresis strategy

Alternatively, a hysteresis strategy is built using two thresholds H_1 and H_2, $H_1 \le H_2$. If the number of customers in the orbit does not exceed H_1, the system operates in the first mode with the input flow rate h_1. If it exceeds H_2, the system operates in the second mode with the input flow rate h_2. When the number of customers is between H_1 and H_2, the system remains in the mode it occupied in the previous moment.

The service process of the system with a hysteresis strategy can be described by a three-dimensional process

$$Q(t, H_1, H_2) =$$
$$= \left(Q_0(t, H_1, H_2), Q_1(t, H_1, H_2), R(t, H_1, H_2)\right),\ t \ge 0,$$

where $Q_0(t, H_1, H_2)$ is the number of busy channels at time t, $Q_1(t, H_1, H_2)$ is the number of repeated attempt sources at time t, $R(t, H_1, H_2)$ is the system operating mode at time t. If $R(t, H_1, H_2) = 1$, then the system operates in the first mode with the input flow rate h_1. If $R(t, H_1, H_2) = 2$, then the system operates in the second mode with input flow rate h_2. The process $Q(t, H_1, H_2) \in S(Q, H_1, H_2)$ Markov chain with continuous time and the phase space

$$S(Q, H_1, H_2) = S^1(Q, H_1, H_2) \cup S^2(Q, H_1, H_2),$$

$$S^1(Q, H_1, H_2) \cap S^2(Q, H_1, H_2) = \varnothing,$$
$$S^1(Q, H_1, H_2) = \{(i, j, 1) : i = 0, 1, ..., c;\ j = 0, ..., H_2\},$$
$$S^2(Q, H_1, H_2) = \{(i, j, 2) : i = 0, 1, ..., c;\ j = H_1 + 1, ...\}.$$

The optimal control problem for the retrial queues with a single retrial attempt consists in searching for such thresholds H_1 and H_2, that are solutions of the optimization problem:

$$L(H_1, H_2) = C_1 L_1(H_1, H_2) + C_2 L_2(H_1, H_2) - C_3 L_3(H_1, H_2)$$
$$- C_4 L_4(H_1, H_2) - C_5 L_5(H_1, H_2) \to \max, \quad (2)$$

$$H_1, H_2 \in \{0, 1, ...\},\ H_1 \le H_2,$$

where the cost coefficients are: C_i is the income relating to the service of a call in the *i*-th mode, $i = 1,2$; C_3 is the penalty for refusing to provide a service; C_4 is the penalty relating to switching of the input flow rate, C_5 is the penalty relating to lost call after failed repeated attempt; $L_i(H_1, H_2) = \lim_{t \to \infty} t^{-1} L_i(t, H_1, H_2)$, $i = \overline{1,5}$: $L_i(t, H_1, H_2)$ is the number of calls served in the *i*-th mode, $i = 1,2$; $L_3(t, H_1, H_2)$ is the number of calls which become repeated calls; $L_4(t, H_1, H_2)$ is the number of switchings of the input flow; $L_5(t, H_1, H_2)$ is the number of calls that leave the system without service after repeated attempt.

For the hysteresis strategy, the quality functional $L(H_1, H_2)$ is built in terms of the system's stationary probabilities (see [4]) according to functionals $L_i(H_1, H_2)$, $i = \overline{1,5}$:

$$L_1(H_1, H_2) = \mu \sum_{j=0}^{H_2} \sum_{i=1}^{c} i \pi_{ij}(1),$$

$$L_2(H_1, H_2) = \mu \sum_{j=H_1+1}^{\infty} \sum_{i=1}^{c} i \pi_{ij}(2),$$

$$L_3(H_1, H_2) = \sum_{j=0}^{H_2} h_1 \pi_{cj}(1) + \sum_{j=H_1+1}^{\infty} h_2 \pi_{cj}(2),$$

$$L_4(H_1, H_2) = h_1 \pi_{cH_2}(1) + (H_1 + 1)\nu \sum_{i=0}^{c} \pi_{iH_1+1}(2),$$

$$L_5(H_1, H_2) = \nu \left(\sum_{j=1}^{H_2} j \pi_{cj}(1) + \sum_{j=H_1+1}^{\infty} j \pi_{cj}(2) \right).$$

The solution to problem (2) is such thresholds H_1 and H_2 that maximize the average profit from the operation of the system.

Conclusion

According to threshold and hysteresis strategies, optimization problems for the effective operation of the system are formulated and solved, while the quality functionals are built in terms of the system's stationary probabilities.

References

[1] J. R. Artalejo, A. Gomez-Corral, "Retrial Queueing Systems: A Computational Approach,". Springer, Berlin, 2008.

[2] G.I. Falin, J.G.C. Templeton, "Retrial Queues," Chapman and Hall, London, 1997.

[3] O.V. Pryshchepa, E.O. Lebedev, "On a Multi-Channel Retrial Queueing System," Cybernetics and Systems Analysis, vol. 53 (3), 2017, pp. 441-449.

[4] O.V. Pryshchepa, "*M/M/c/N*-type system with single retrial attempt, controlled by a hysteresis strategy," Bulletin of the Taras Shevchenko National University of Kyiv. Series: Physical and Mathematical Sciences, No. 4, 2011, pp. 193–198.

[5] O. Pryshchepa, O. Kardash, A. Yakymchuk, M. Shvec, K. Pavlov, O. Pavlova, I. Irtyshcheva, N. Popadynets, Ye. Boiko and I. Kramarenko, "Optimization of multi-channel queuing systems with a single retrail attempt: Economic approach," Decision Science Letters, vol. 9(4), 2020, pp. 559–564.

[6] Y.W. Shin and D.H. Moon, "Approximations of retrial queue with limited number of retrials," Computers and Operations Research, vol. 37, №7, 2010, pp. 1262-1270.